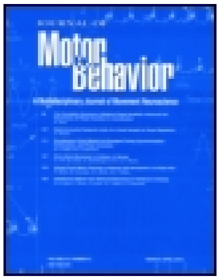

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To Pass or Not to Pass: A Mathematical Model for Competitive Interactions in Rugby Union

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RESEARCH ARTICLE

To Pass or Not to Pass: A Mathematical Model for Competitive Interactions in Rugby Union

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ABSTRACT. Predicting behavior has been a main challenge in human movement science. An important step within the theory of coordination dynamics is to find out the rules that govern human behavior by defining order parameters and control parameters that support mathematical models to predict the behavior of a system. Models to describe human coordination have been focused on interlimb coordination and on interpersonal coordination in affiliative tasks but not on competitive tasks. This article aims to present a formal model with two attractors to describe the interactive behavior on a 2v1 system in rugby union. Interpersonal distance and relative velocity critical values were empirically identified and were included as task constraints that define the attractor landscape. It is shown that using relative velocity as a control parameter the model offers reasonable prediction concerning the decision-making process. The model has the plasticity to adapt to other settings where interpersonal distances and relative velocities amongst system components act as significant task constraints.

Keywords: decision making, dynamical system, interpersonal coordination, stable state

The core of this article is to present a low-dimensional description of fundamental processes that are implicit to the interactions between high-dimensional biological systems (i.e., players; J. A. S. Kelso, 1988; S. Kelso, 1995; Newell, Liu, & Mayer-Kress, 2008; Schoner & Kelso, 1988), with a formal model with two attractors that roughly describe the interaction between two attackers and one defender in rugby union. The three agents are coupled throughout by contextual information, such as continuous changes on interpersonal distances and relative velocities, which influence players' decisions and actions in the neighborhood. This contextual dependency implies that three players in a 2v1 situation in rugby union behave as a nonlinear dynamical system. There are a number of difficulties involved in testing dynamical systems properties in natural settings, such as the manipulation of hypothetical control parameters and the analysis of changes in the order parameters. Therefore, it is the purpose of the present work to describe a 2v1 system in rugby union as a dynamical system with two attractors, in order to explain why a three agent system remains as a three agent system (i.e., a 2v1 situation) or why it is attracted to a two competitive agent system (i.e., a 1v1 situation).

Characterization of the 2v1 System

The 2v1 system is a typical situation in rugby union that can be roughly described as a situation in which a ball carrier aims to commit the defender with him/her, usually running toward the defender and then passing the ball to a support player. The support player needs to manage the interpersonal distance to the ball carrier, being available to receive the ball and to run free toward the goal line. The defender aims (a) to tackle the ball carrier, (b) to intercept the ball, or (c) to tackle the support player following the reception of the ball (Figure 1). Grounded on both players' relative position, the defender can adopt various strategies to realize these goals. For instance, the defender can run fast toward the ball carrier aiming to tackle him or her (expressed by a fast decrease on the ball carrier–defender interpersonal distance) or the defender can choose to maintain a relative position to the ball carrier while he or she is running forward on the pitch, aiming to drive the attacker to the sideline and tackle him or her closer to the score line (known as a drifted defense).

In typical 2v1 conditions, there are (at least) two possible final states to where the system is attracted, on the basis of the ball carrier behavior: (a) the ball is passed to the support player; (b) the ball carrier decides to go forward for a 1v1 situation. Nevertheless, we recognize the existence of other possible states that will be discussed as plausible model predictions in the Discussion section. By states, we mean attractive states of the collective variable (i.e., order parameter) dynamics, signifying that the collective variable converges in time to a limited set of solutions (e.g., the ball carrier passes the ball to the support player or the ball carrier decides to go forward; J. A. S. Kelso & Engstrom, 2006; S. Kelso, 2009). In both outcomes, the defender neither intercepts the ball nor tackles the ball carrier; therefore, the attackers always succeed.

Beyond speculations, the reasons that lead the ball carrier to pass the ball to the support player or to go forward remain unknown. However, recent empirical research grounded on robust observation techniques claims that players' interpersonal distances and relative velocities may help to understand why the system is attracted to one of the two states (Passos, Cordovil, Fernandes, & Barreiros, 2012). In fact, the analysis of a random sample of trials selected from a total of 65, showed that around the moment of the ball carrier decision two distinct situations happened: (a) when the ball carrier–defender interpersonal distance was small and the ball carrier–defender relative velocity was negative (i.e., the defender velocity was increasing and

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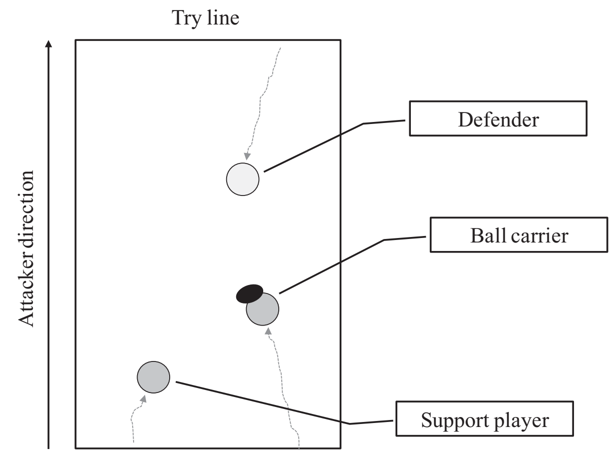

FIGURE 1. Schematic representation of a 2v1 system in rugby union.

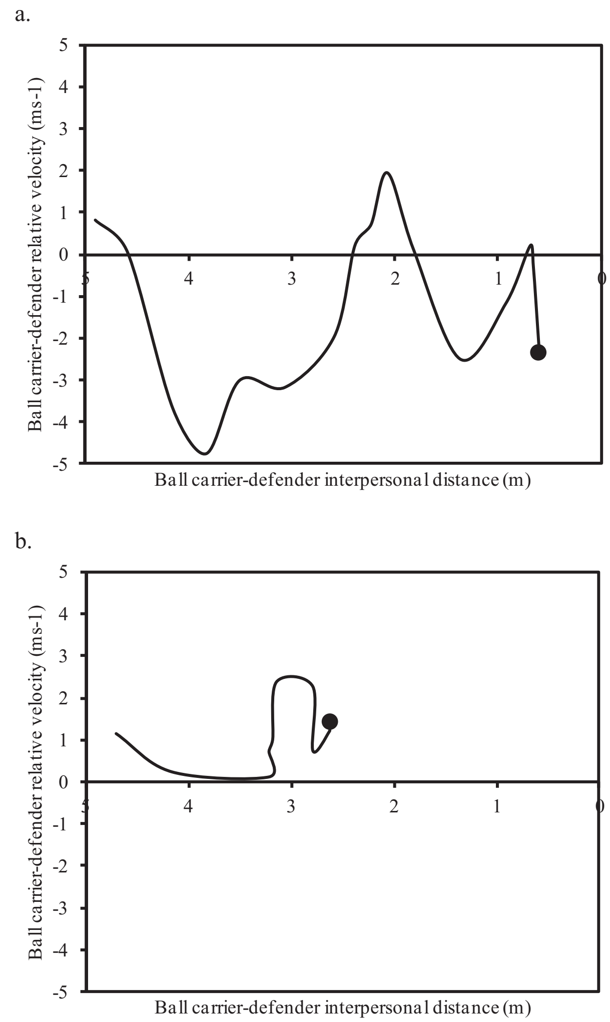

FIGURE 2. Graphical representation of the ball carrier–defender relative velocity versus the ball carrier–defender interpersonal distance: a. outcome passes the ball; b. outcome goes forward.

the ball carrier velocity was decreasing or stable), the ball carrier decided to pass the ball to the support player; (b) when the ball carrier–defender interpersonal distance was large and the ball carrier–defender relative velocity was positive (i.e., the ball carrier velocity was increasing whereas the defender velocity was decreasing or stable), the ball carrier decided to go forward. More precisely, for negative relative velocities the percentage of times a pass occurred was 80.0%, whereas for positive relative velocities the percentage of times it occurred was 14.3% ($p = 0.01$); on the other hand, for positive relative velocities the percentage of times a go forward took place was 85.7%, while for negative relative velocities the percentage of times it took place was 20.0% ($p = 0.01$). Regarding the trials where the relative velocities were around zero, namely in the neighborhood of zero with length of one standard deviation, the percentage of times a pass occurred was 60.0% and the percentage of times a go forward took place was 40.0%. Figure 2a displays an example of a trial where the pass occurred and Figure 2b exhibits an example of a trial where the ball carrier went forward. For both figures the data show relative velocity fluctuations and interpersonal distances until the moment of the decision (black circle), namely a pass below 1 m of interpersonal distance with negative relative velocity (Figure 2a) and a go forward close to 3 m of interpersonal distance with positive relative velocity (Figure 2b). Therefore, to describe the system behavior at the moment of the ball carrier decision, we will use one order parameter—the ball carrier–defender interpersonal distance—which may assume two possible conditions: (a) the order parameter assumes lower values, which is consistent with the ball carrier decision to perform a pass to the support player; (b) the order parameter assumes higher values, which is consistent with the ball carrier decision to go forward. Moreover, we will additionally include one control parameter: the ball carrier–defender relative velocity, which may take negative or positive values. The pertinence of a formal model to describe the players' behavior in the above mentioned conditions depends on the plausibility of this control parameter and its contribution to drive the system to one of the two alternative outcomes (Figure 3).

Task Constraints and Control Parameters

Previous studies in rugby union revealed interesting results that are useful to highlight the task constraints and to indicate the best candidate to act as a control parameter in the model. In team sports, the ball carrier actions are closely linked to the interpersonal distance to the defender. Interpersonal distances were previously characterized as

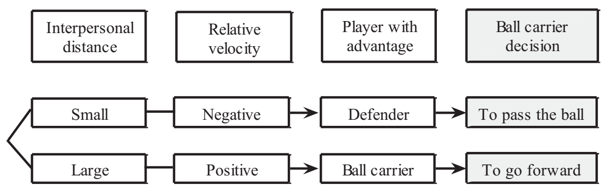

FIGURE 3. Schematic representation of the ball carrier–defender interpersonal distance and the ball carrier–defender relative velocity in the outcomes passes the ball and goes forward.

critical regions, where players' behaviors are constrained by a contextual dependency (Passos et al., 2009). In these critical regions, the ball carrier chooses between performing a pass to the support player and to go forward. Outside the critical regions, the system tends to keep a certain structural organization, maintaining the attacker–defender balance, which means that the defender keeps a position between the attackers and the goal line. To cross the defender's line the ball carrier must enter the critical region in which a decision has to be made concerning the decision to pass the ball or to remain with the ball and go forward in a 1v1 scenario. Outside the critical regions, the decision-making process may well be driven by general behavioral prescription, such as strategic organization and team structure, but inside the critical regions the effect of some control parameter has to be considered. For instance, the relative velocity—the difference between the attacker velocity and the defender velocity—is a promising parameter to explain why the attacker–defender dyad evolves to a certain outcome (i.e., a defender tackle or an attacker try). The literature suggests that the decreasing of opponent players' interpersonal distance drives the attacker–defender dyadic system into a critical region. Within the critical region, the relative velocity plays a major role as a control parameter. Research about attacker–defender dyads in rugby union reveals that, at interpersonal distances smaller than 4 m, when players are running toward each other, the player that is increasing velocity will gain advantage over the other (Passos et al., 2008). It is also known that in order to diminish any odds of the defender to tackle the support player, the pass should be performed below 1 m of interpersonal distance (Passos et al., 2012). The 1 m boundary was also observed in collision avoidance between walkers (Olivier, Marin, Crétual, & Pettré, 2012), indicating that collision avoidance in humans may happen at distances around this critical value. We are aware, however, that this critical value may well be dependent on skill level and other factors.

For the proposed model, and within a critical region of less than 4 m of interpersonal distance, the ball carrier–defender relative velocity will be considered as a suitable control parameter. To specify the relative velocity at the moment of the decision, we will also consider 5 ms^{-1} as a reasonable maximum value of the relative velocity (Hendricks, Karpul, Nicolls, & Lambert, 2012; Passos et al., 2008).

Modeling the General Features of Players' Interactions

The aim of the proposed formal model is to capture whether the interactions between two attackers and one defender evolves into a 2v1 situation or falls into a dyadic 1v1 situation. In this case, there is an order parameter and two reproducibly observed states (i.e., to pass the ball or to go forward) that can be characterized as attractors, or stable states, to where the system moves. Prior research on the dynamics of bimanual coordination showed that when subjects start to cycle their index fingers in an antiphase mode, an increase in the cycling frequency generates a switch to an inphase pattern (J. A. S. Kelso, 1984). The experimental observations revealed important properties of self-organization, such as phase transitions, multistability, and hysteresis. These empirical results led to a hallmark theoretical (HKB) model with two attractors and transitions from one attractor to the other at specific values of movement frequency, that in turn was derived from nonlinear interactions among the moving components (Haken, Kelso, & Bunz, 1985). Combining concepts of synergetics (e.g., order parameters–collective variables, control parameters, slaving principle) and tools of nonlinear dynamical systems (e.g., instability, multistability, timescales), the emergent behavior of the system was formally modeled using a potential function and a differential equation (motion equation), which offers a mathematical description of the system's behavior as time passes and parameters change. The potential function describes an attractor landscape, in which the valleys function as attractors and reflect relatively stable behavioral states. If the order parameter is denoted by x and the potential function is denoted by V, then the evolution of x over time can be expressed by the differential equation $dx/dt = -dV/dx$. A value of x for which the derivative dx/dt is equal to zero corresponds to a steady state, with a minimum of V signaling an attractor and a maximum of V a repeller. In the HKB model, the order parameter was the relative phase φ of the oscillators and the potential function V, which was presumed to be periodic and symmetric, was defined as $V(\varphi) = -a\cos(\varphi) - b\cos(2\varphi)$, where a and b were two control parameters. Thus, the motion equation was written as $d(\varphi)/dt = -a\sin(\varphi) - 2b\sin(2\varphi)$ (by rescaling, the parameters a and b were then expressed as a single parameter $k = b/a$).

Beyond symmetric interlimb coordination, the HKB model was later extended to numerous coordination tasks. J. A. S. Kelso, DelColle, and Schöner (1990) established a broken symmetry version of the model for sensorimotor coordination (Kelso et al., 1990). This generalized model handled the asymmetry between the oscillators by including a detuning term defined as the arithmetical difference between the uncoupled frequencies. In another study of interpersonal coordination pairs of participants were instructed to perform a rhythmic task of visual coordination of their outer legs (Schmidt, Carello, & Turvey, 1990). The results displayed two modes of coordination, antiphase and inphase, where the switching between these two modes of coordination was due to changes on the control parameter values, namely the frequency of oscillation of the legs.

However, despite some general and useful features (e.g., describing a system behavior with two attractors), the HKB model is not suitable for 2v1 systems in rugby union, because it is a cyclic model based on trigonometric functions. Therefore, our model is close to previous work on the dynamics of speech perception that showed that listeners perceive the word *say* at short silent gaps (between the "s" and the "ay") and perceive the word *stay* at long silent gaps reflecting properties of self-organization (Tuller, Case, Ding, & Kelso, 1994). The empirical results motivated a formal model with two attractors and transitions from one attractor to the other at particular values of gap duration (Tuller, Ding, & Kelso, 1997). In the speech perception model, the order parameter x was a variable characterizing the perceptual form and the potential function V was written as $V(x) = k\,x - x^2/2 + x^4/4$, where k was a control parameter. So, the motion equation was written as $dx/dt = -k + x - x^3$.

It is worth mentioning that the order and the control parameters for the HKB and the Tuller models are clearly identified and can thus capture the systems dynamics in terms of mathematically defined attractors (Kijima et al., 2012). However, we experience an increased difficulty when attempting to apply these models to team sports interactive behaviors. To overcome this difficulty it is necessary to extract the most relevant variables that describe the dynamics of the dyadic system behavior. Although this is a nontrivial matter, it is worth noting that Tuller et al.'s (1997) model was already used to describe several systems as dynamical systems with two attractors in sport tasks. Araújo, Davids, and Hristovski (2006) investigated sailing regattas, and found that the place chosen to start, on the right or left-hand side of the starting line, changed under the influence of a given parameter. In this work, the order parameter was expressed by the place on the starting line where the sailors started and the control parameter was the angle between the wind direction and the starting line (Araújo et al., 2006). Some generalizations of Tuller et al.'s model were also used for the description of dynamical systems with more than two attractors. Araújo, Diniz, Passos, and Davids (2014) proposed three possible coordination patterns characterized as three stable attractors for attacker–defender dyads in rugby union: clean try, effective tackle, and tackle where the attacker passes the defender. Here, the order parameter was the angle between the attacker–defender vector and an imaginary horizontal line parallel to the try line and the control parameters were the relative velocity and the interpersonal distance (Araújo, Diniz, Passos, & Davids, 2014).

In our proposed model for 2v1 systems, the candidate order parameter is related to the ball carrier–defender interpersonal distance, with the constraint that the actions of interest are performed within a certain critical region (i.e., short interpersonal distance between ball carrier and defender). This choice was grounded on a qualitative criterion, based on the different paths displayed by the ball carrier–defender interpersonal distance in each ball carrier decision (i.e., to pass the ball or to go forward). In this model, it is assumed that the interpersonal distance only takes values between 0 and 4 and so the variable two minus the interpersonal distance takes values between –2 and 2. Therefore, for symmetry reasons, the candidate order parameter is equal to two minus the interpersonal distance from the ball carrier to the defender. If the attractor layout is the set of possible outcome behaviors of the system, then the changes on this layout are due to changes in the system organization. We assume that if the ball carrier–defender interpersonal distance is characterized by fluctuations, that is a consequence of a pass from the ball carrier to the support player, which means that the system is attracted to a 2v1 situation—we name it attractor 1; if the ball keeps a path characterized by a continuous decreasing of ball carrier–defender interpersonal distance, then the system is attracted to a 1v1 situation—we name it attractor 2. As previously stated, the control parameter is the ball carrier–defender relative velocity: (a) if decreasing (or remaining), the ball carrier passes the ball; (b) if increasing, the ball carrier goes forward. In the first case, for example, the relative velocity decreases due to a decrease of the ball carrier running speed, or the relative velocity remains the same due to a decrease of both players running speed.

Formally, if the order parameter of the system—two minus the ball carrier–defender interpersonal distance—is denoted by x, then the behavior of the system is defined by changes in x over time and can be expressed by the following differential equation

$$\frac{dx}{dt} = -\frac{dV}{dx},$$

where V is a potential function, which describes the attractor landscape. Based on the system dynamics, the following function V seems suitable

$$V(x) = \frac{x^4}{4} - \frac{x^2}{2} + kx, \quad (1)$$

where x lies between –2 and 2 and k is a control parameter that, over a certain range of values, namely k between $-k_m$ and k_m, takes the system from one stable state to another. Thus, the motion equation can be written as

$$\frac{dx}{dt} = -x^3 + x - k. \quad (2)$$

Due to the inherent variability of the system, there are also random fluctuations that cannot be measured. In mathematical terms, we suggest to model these fluctuations as random white noise, which leads to the more general equation

$$\frac{dx}{dt} = -x^3 + x - k + e_t,$$

where e_t is a white noise with mean zero and variance Q. The effect of this stochastic process on the behavior of the system depends on the magnitude of Q (Schoner, Haken, & Kelso, 1986). The occurrence of random fluctuations, namely pink noise (i.e., one over f noise), has also been observed in studies of sensorimotor coordination (Treffner & Kelso, 1999). More recently, the presence of this kind of fluctuations, from white noise to pink noise, has been reported in several human movement studies (Diniz, Barreiros, & Crato, 2010, 2012; Diniz et al., 2011).

To account for the features observed with empirical data, the control parameter k must be a function of the relative velocity v. Based on empirical values, we propose the following equation

$$k = k(v) = \left(\frac{v}{v_m/r}\right)^3, \tag{3}$$

where v_m is the maximum relative velocity which means that v lies between $-v_m$ and v_m and r is a normalizing constant given by $r = \sqrt[3]{k_m}$ so that, when v goes from $-v_m$ to v_m, k goes from $-k_m$ to k_m. A frequency distribution of experimental observations based on the percentage of outcomes (i.e., to pass the ball or to go forward) for several ranges of relative velocity v motivates this functional form of the control parameter k (Passos et al., 2012). In this particular situation, to ensure that the attractors are located between –2 and 2, the control parameter k must lie between –6 and 6 and thus $k_m = 6$ and $r = \sqrt[3]{6}$. On the other hand, it is reasonable to suppose that the relative velocity v lies between -5 and 5 and so $v_m = 5$ (Hendricks et al., 2012; Passos et al., 2008).

Results

The existence of critical points beyond which the system is pulled to one of the two attractors is very important. In order to calculate the critical values x_c, k_c, and v_c at which a transition to one of the attractors occurs, it is necessary to study the extremes of V (i.e., the roots of dV/dx). The third-order polynomial dV/dx has one, two, or three (real) roots. If one of the roots of the polynomial dV/dx is denoted by x_r, then the polynomial can be written as

$$\begin{aligned}\frac{dV}{dx} &= x^3 - x + k \\ &= (x - x_r)(x^2 + x_r x + x_r^2 - 1).\end{aligned}$$

Now consider the second-order polynomial P given by

$$P(x) = x^2 + x_r x + x_r^2 - 1.$$

The possible roots of the polynomial P can be written in the form

$$x = \frac{-x_r \pm \sqrt{-3x_r^2 + 4}}{2}.$$

The system evolves toward one of the two attractors when the polynomial dV/dx has only one root, which happens whenever the polynomial P has no roots. This is true when the former two roots vanish, which happens provided

$$-3x_r^2 + 4 < 0 \Leftrightarrow x_r^2 > \frac{4}{3} \Leftrightarrow |x_r| > \sqrt{\frac{4}{3}}.$$

In sum, the transition to one of the attractors occurs at critical values x_c such that

$$|x_c| = \sqrt{\frac{4}{3}} \approx 1.15,$$

and the corresponding critical values k_c satisfy

$$|k_c| = |x_c|^3 - |x_c| \approx 0.38.$$

The corresponding critical values v_c with $v_m = 5$ and $k_m = 6$ satisfy

$$|v_c| = \frac{v_m}{\sqrt[3]{k_m}} \sqrt[3]{|k_c|} \approx 2.00.$$

Figure 4 displays the graph of the potential function (on the left) defined by Equation 1, the motion equation (on the center) expressed by Equation 2, and the control parameter function (on the right) given by Equation 3, for seven values of the control parameter k (i.e., k = –2.0, –1.0, –0.38, 0.0, 0.38, 1.0, 2.0). On the graphs of each potential function and each motion equation the value of the control parameter k is shown as well as the values of the order parameter x corresponding to the extremes of the potential function V (i.e., the roots of the polynomial function dx/dt (= –dV/dx)) (in parentheses the value of the equivalent ball carrier–defender interpersonal distance d is shown). On the graph of each control parameter function the value of the control

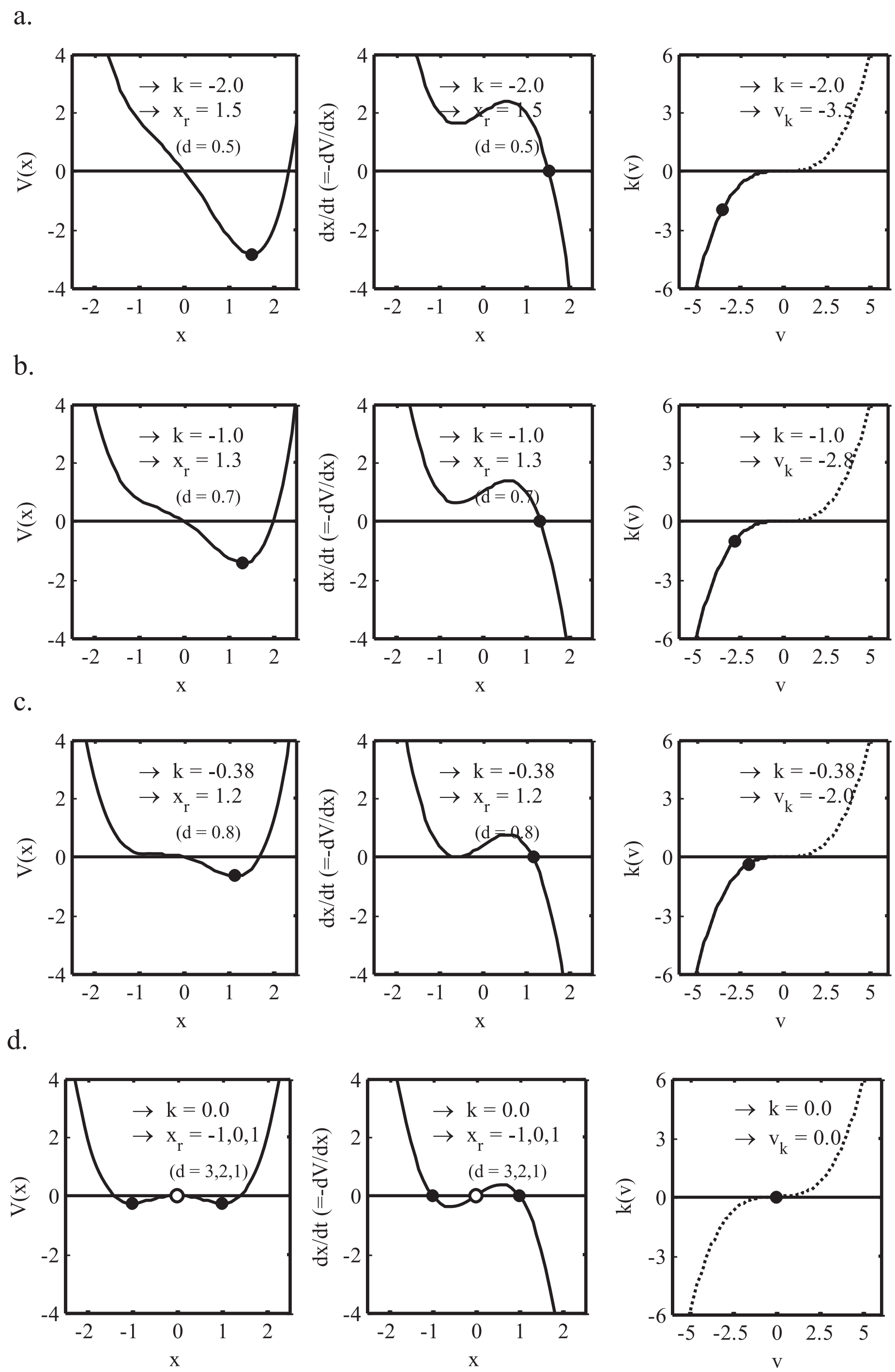

FIGURE 4. Graphical representation of the potential function (on the left), the motion equation (on the center), and the control parameter function (on the right) for seven values of the control parameter k: a., b., c. k = –2.0, k = –1.0, k = –0.38, respectively, attractor 1 passes the ball; d. k = 0.0 bistability; e., f., g. k = 0.38, k = 1.0, k = 2.0, respectively, attractor 2 goes forward.
(*Continued*)

parameter k is presented as well as the value of the corresponding relative velocity v.

If the value of the order parameter x at which the minimum of the potential function V occurs is positive and larger than the positive critical value, then the value of the control parameter k is negative and thus the value of the relative velocity v is also negative. This means that the ball carrier–defender interpersonal distance is small and the

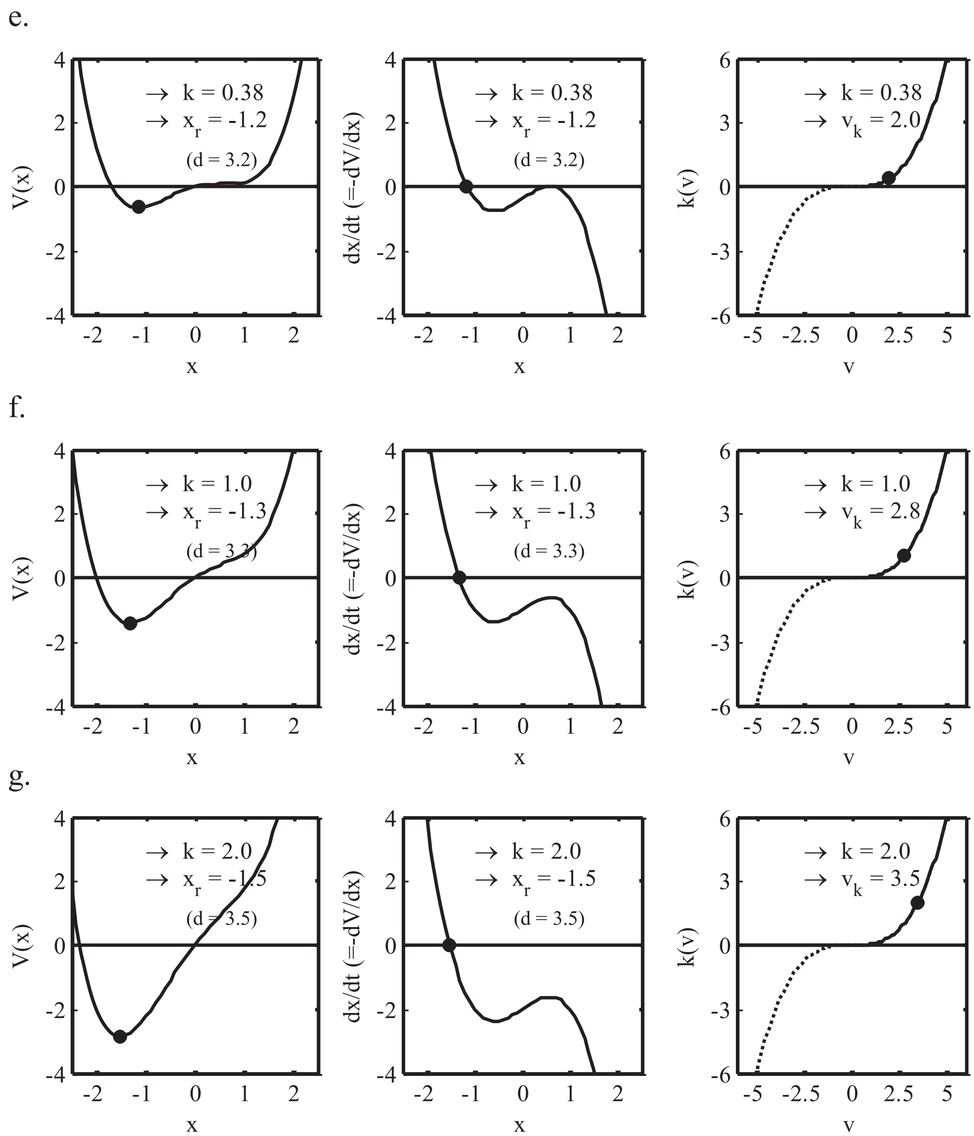

FIGURE 4. (Continued)

defender velocity is larger than the ball-carrier velocity, leading the system to attractor 1 (i.e., the ball carrier passes the ball). In contrast, if the value of the order parameter x at which the minimum of the potential function V occurs is negative and smaller than the negative critical value, then the value of the control parameter k is positive and therefore the value of the relative velocity v is also positive. This means that the ball carrier–defender interpersonal distance is large and the ball-carrier velocity is larger than the defender velocity, leading the system to attractor 2 (i.e., the ball carrier goes forward). Interestingly, when the values of the order parameter x, corresponding to the extremes of the potential function V, lie between the negative critical value and the positive critical value, situations of bistability take place. This reflects the presence of an equidistribution of the relevant variable (namely at $k = 0.0$), reproducing the uncertainty of the decision-making in those situations. The empirical observations are mainly in concordance with these predictions of the model. More precisely, a percentage of 86.0% of the trials is predicted by the model (Passos et al., 2012).

Discussion

The proposed model predicts that whenever the defender is faster than the ball carrier, the system evolves toward attractor 1, which means that the ball carrier will perform a pass to the support player. The model also shows that the decision of performing a pass happens at very short interpersonal distances between the ball carrier and the defender. This distance is in accordance to empirical data concerning collision avoidance in human walkers (Olivier et al., 2012) and with data from a previous investigation of the 2v1 situation in rugby union (Passos et al., 2012). On the contrary, whenever the ball carrier is faster than the defender, the system evolves to a 1v1 situation, which is consistent with attractor 2. The model also shows that the decision to go forward occurs at larger ball carrier–defender interpersonal distances.

The system has two alternative states, the time evolution of which is shown to depend on a single control parameter: the relative velocity of the ball carrier and defender. An interesting finding is that the shifting points that define that the system evolves to one of the two alternative states correspond exactly to relative velocities of ± 2 ms^{-1}. In the context of rhythmic movements, the transitions between two states induced by the scaling of a control parameter, such as limb frequency, have been demonstrated (J. A. S. Kelso, Schoner, Scholz, & Haken, 1987). Also, the decision-making processes in team sports have highlighted the role of interpersonal distance and relative position of players in what concerns the formation of patterns in ecological conditions (Vilar, Araujo, Davids, & Travassos, 2012) and other variables like the proximity-to-goal (Headrick et al., 2012).

In further work, we plan to examine the influence of other possible control parameters besides relative velocity, for instance, angular deviation. It may even be the case that the interaction between these quantities is important (e.g., when angular deviation decreases as well as increases).

An important characteristic of this model is that it can be tested in any social context whereas participants' interactive behavior evolves toward two mutually exclusive outcomes. For instance, it can be used in other team sports games rather than rugby or in social settings such as traffic jam (i.e., to overtake or not to overtake the car in front of me). These are only a few examples of clues for further research that can be explored from this 2v1 model. But one of the most relevant features of this model is that decisions and actions made based upon affordance properties can be grounded in the dynamics of coordination. Of additional relevance is that this model is a good example of the dynamics involved in the switching of behavioral modes. Whereas there are plenty of examples of behavioral steady state dynamics, there are only a few examples of such dynamical phase transitions that capture the switching of behavioral modes in such a real life complex example of coordination. One important example of such dynamical switching of behavioral modes concerns the description of an attacker–defender dyad in rugby union as a dynamical system with three attractors: clean try, effective tackle, and tackle where the attacker passes the defender (Araújo et al., 2014). In this case, under the influence of two control parameters, the dyadic system is attracted to one of the three coordinated states.

Next, we conclude our discussion by drawing attention to some further effects likely in 2v1 situations in rugby union, that due to lack of empirical data were not included in this model, namely the dummy pass to deceive the defender, the off load pass due to hard contact between ball carrier and defender, and the grubber kick, where the ball is kicked along the ground past the defender and (potentially) gathered again by the attacker.

The dummy pass is a situation where the ball carrier goes forward. More precisely, the dummy is a fake movement performed by the ball carrier, that aims to deceive the defender, make him/her change the relative position and consequently open a gap to afford the ball carrier to go forward toward the try line. This deceptive movement is performed at close interpersonal distance to the defender, which means a low order parameter value; otherwise the defender is able to recover the initial position. Also, it is expected that, if properly executed, causes uncertainty in the potentially decreasing defender velocity, meaning positive values of relative velocity. This prediction highlights the need to test the relative angle between ball carrier and defender as a candidate control parameter. It is predictable that the deceptive movement, which characterizes a dummy pass changes the angle and the relative velocity values, which under certain (unknown) threshold values, affords the ball carrier to go forward at very close interpersonal distance to the defender. But a dummy at very close interpersonal distance values will also afford a tackle for the defender, meaning that to perform a dummy there is also a collision boundary on the interpersonal distance.

A further possibility is the off load pass related to the existence of contact between ball carrier and defender. In this situation, two outcomes may occur: (a) the relative velocity is positive and the ball carrier defeats the defender and goes forward; or (b) the relative velocity is equal or below zero and the defender is in advantage, unless the support player strips the ball and goes forward. This is one of the hardest situations to model, because when the contact happens other variables must be considered, such as the players body mass and center of mass, the point of contact between them, the relative angle between the players at the moment of the contact, and the distortion of the soft tissues, just to name a few. More situations to be considered suggests more complexity and may increase the difficulty to model such systems. Indeed, one of the messages of coordination dynamics is that adding more and more variables may not necessarily enhance insight. The key rather is to identify situations, such as those studied here, where relevant collective variables and control parameters can be empirically identified and mapped onto a theoretical model from which further predictions may be tested.

In rugby union, another way to go forward and conquer territory to the opponent is playing with the feet. One common skill is the grubber kick, where the ball carrier kicks the ball forward aiming to achieve a path close to the ground. We hypothesize that for a grubber kick to succeed, the relative velocity values must be positive (i.e., at the moment of the kick, the ball carrier is running faster than the defender), and the interpersonal distance (i.e., the order parameter) has a critical threshold. If the grubber kick is performed above this threshold, it affords both players to run and grab the ball from the floor; if performed below the threshold, the relative velocity behaves as a control parameter. After the kick, both players will run toward the ball, which is placed behind the defender. Thus, the interpersonal distance threshold can be modeled based on the players' acceleration profile, which defines how long each one takes to run a certain distance.

Finally, a common issue that arises in the context of 2v1 situations concerns the possibility of combining candidate control parameters (e.g., in dynamical models of codimension two or higher; Thom, 1976) which afford much more complex behavior. In the current model the combination of relative angles thresholds with velocity and acceleration profiles seems interesting. A further matter that was not considered in the present model is the participants' skill level. To fill this gap in further studies, we suggest modeling the skill level based on players' speed. We can presume that, concerning decisions and actions, skilled players are faster players. Thus, we can model the 2v1 with significant speed differences between ball carrier and defender, which consequently will provoke changes in the critical regions characterized with ball carrier–defender interpersonal distances. Our hypothesis is that, when the ball carrier is faster (i.e., more skilled) than the defender, the critical regions will decrease (i.e., the ball carrier decisions and actions will happen at very short interpersonal distances); on the other hand, when the ball carrier is not so fast (i.e., not so skilled), the critical regions will display a higher length. A final subject is the prospect of modeling the 2v1 considering previous success or failure sustained on risk behavior. After a failure, players display less risky behaviors, and following a success, risky behaviors are more likely to happen. Our hypothesis here is that, after a failure, the ball carrier decisions will occur at higher interpersonal distances from the defender, increasing the length of the critical regions; in contrast, a success will lead the players to increase the risk of decisions, decreasing the length of the critical regions. All these issues, and more, present formidable future challenges to understanding human interactive behavior in competitive sports situations.

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